Inspection Games over Time
Fundamental Models and Approaches
Rudolf Avenhaus, Thomas Krieger
Inspection Games over Time
Fundamental Models and Approaches

Rudolf Avenhaus, Thomas Krieger
Contents

Foreword 1
Preface III

1 Introduction 1
1.1 Classification of assumptions for inspection models over time . . . . . . . . . . 2
1.2 Applications . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 5
1.3 The art of modelling . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 6
1.4 Structure of the monograph . . . . . . . . . . . . . . . . . . . . . . . . . . . 8
1.5 Who is the client? . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10

1 Playing for Time: Discrete Time 13

2 General assumptions 15

3 No-No inspection game 21
3.1 Any number of inspection opportunities and one interim inspection . . . . . . . 21
3.2 Special numbers of inspection opportunities and two interim inspections . . . 41

4 No-Se and Se-No inspection games 49
4.1 No-Se for special numbers of inspection opportunities and interim inspections . . 50
4.2 Se-No for any number of inspection opportunities and interim inspections:
    Krieger-Avenhaus model . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 54

5 Se-Se inspection game 75
5.1 Any number of inspection opportunities and interim inspections . . . . . . . 75
5.2 Any number of inspection opportunities and interim inspections:
    a recursive approach . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 83
5.3 Any number of inspection opportunities and interim inspections: step by step inspection game ........................................... 91

6 Models with errors of the second kind ........................................... 99
   6.1 No-No: Any number of inspection opportunities and one interim inspection; three inspection opportunities and two interim inspections .................. 100
   6.2 No-Se: Three inspection opportunities and two interim inspections .......... 113
   6.3 Se-No: Any number of inspection opportunities and one interim inspection; three inspection opportunities and two interim inspections .................. 117
   6.4 Se-Se: Three inspection opportunities and two interim inspections ............ 124
   6.5 Comparison of variants ..................................................... 128
   6.6 Applications to Nuclear Safeguards ........................................ 129

7 Legal behaviour, effectiveness and efficiency, extensions ..................... 133
   7.1 Historical development ..................................................... 134
   7.2 Utilities for attribute sampling inspection schemes ................................ 135
   7.3 Effectiveness and efficiency ................................................ 143
   7.4 Utilities for variable sampling inspection schemes and further extensions .... 145

II Playing for Time: Continuous Time ............................................ 149

8 General assumptions ......................................................... 151

9 No-No inspection game: Diamond model and extensions ..................... 155
   9.1 One interim inspection ..................................................... 156
   9.2 Any number of interim inspections ........................................ 161
   9.3 Any number of interim inspections; errors of the second kind .................. 169
   9.4 One interim inspection; errors of the first and second kind ....................... 172
   9.5 Choice of the false alarm probability ...................................... 178

10 Se-No inspection game for one facility: Avenhaus-Krieger model ............ 185
   10.1 Any number of interim inspections; errors of the second kind .................. 186
   10.2 Applications to Nuclear Safeguards ........................................ 198
   10.3 Two interim inspections; errors of the first and second kind ................... 200

11 Se-No inspection game for more facilities: Krieger-Avenhaus model ........... 207
   11.1 One interim inspection; facility-dependent errors of the second kind .......... 208
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.2</td>
<td>Two interim inspections; facility-independent errors of the second kind</td>
<td>214</td>
</tr>
<tr>
<td>11.3</td>
<td>Any number of interim inspections; facility-independent errors of the second kind</td>
<td>222</td>
</tr>
<tr>
<td>12</td>
<td>Se-Se inspection game: Avenhaus-Canty model</td>
<td>233</td>
</tr>
<tr>
<td>12.1</td>
<td>One interim inspection; errors of the first and second kind</td>
<td>235</td>
</tr>
<tr>
<td>12.2</td>
<td>Two interim inspections; errors of the first and second kind</td>
<td>240</td>
</tr>
<tr>
<td>12.3</td>
<td>Any number of interim inspections; errors of the first and second kind</td>
<td>252</td>
</tr>
<tr>
<td>12.4</td>
<td>Choice of the false alarm probability</td>
<td>261</td>
</tr>
<tr>
<td>13</td>
<td>Comparison of models in Part II and between models in Parts I and II</td>
<td>265</td>
</tr>
<tr>
<td>III</td>
<td>Critical Time</td>
<td>273</td>
</tr>
<tr>
<td>14</td>
<td>General assumptions</td>
<td>275</td>
</tr>
<tr>
<td>15</td>
<td>No-No inspection game: Canty-Rothenstein-Avenhaus model</td>
<td>281</td>
</tr>
<tr>
<td>15.1</td>
<td>Two periods and one inspection; errors of the first and second kind</td>
<td>282</td>
</tr>
<tr>
<td>15.2</td>
<td>Three periods and two inspections; errors of the first and second kind</td>
<td>286</td>
</tr>
<tr>
<td>15.3</td>
<td>Any number of periods and inspections; errors of the first and second kind</td>
<td>293</td>
</tr>
<tr>
<td>15.4</td>
<td>Sensitivity considerations</td>
<td>307</td>
</tr>
<tr>
<td>15.5</td>
<td>Choice of the false alarm probability</td>
<td>311</td>
</tr>
<tr>
<td>16</td>
<td>Se-Se inspection game: Dresher-Höpfinger model and extensions</td>
<td>315</td>
</tr>
<tr>
<td>16.1</td>
<td>Any number of steps and controls</td>
<td>316</td>
</tr>
<tr>
<td>16.2</td>
<td>Any number of steps and controls; errors of the second kind</td>
<td>327</td>
</tr>
<tr>
<td>16.3</td>
<td>Any number of steps and one control; errors of the first and second kind</td>
<td>337</td>
</tr>
<tr>
<td>16.4</td>
<td>Choice of the false alarm probability</td>
<td>351</td>
</tr>
<tr>
<td>17</td>
<td>Strait control and models with multiple illegal activities</td>
<td>353</td>
</tr>
<tr>
<td>17.1</td>
<td>Any number of nights and controls; errors of the second kind: Generalized Thomas-Nisgav model, Models by Baston and Bostock and by Garnaev</td>
<td>354</td>
</tr>
<tr>
<td>17.2</td>
<td>Multiple illegal activities: Models by von Stengel, by Sakaguchi, by Ferguson and Melolidakis, and by Hohzaki</td>
<td>376</td>
</tr>
<tr>
<td>18</td>
<td>Classification of models in Part III</td>
<td>389</td>
</tr>
</tbody>
</table>
IV Appendixes 393

19 Non-cooperative two-person games 395
  19.1 Normal form games ........................................... 395
  19.2 Extensive form games ........................................ 400

20 Receiver Operating Characteristic of a binary classifier system 403

21 Proof of Lemma 10.3 407

22 Recurrence relations used in Chapters 10, 11 and 12 411

23 Supplementary considerations to Sections 11.2 and 11.3 415
  23.1 Proof of (11.41) .............................................. 415
  23.2 Lemmata for the proof of Theorem 11.2 ...................... 417

24 A Se-No inspection game with an expected number of inspections:
   Krieger-Avenhaus model 425
  24.1 Three steps; errors of the second kind ...................... 427
  24.2 Any number of steps; errors of the second kind ............ 433

List of Figures 439

List of Tables 445

References 449
Inspection Games over Time
Fundamental Models and Approaches

Rudolf Avenhaus, Thomas Krieger